AP Calculus BC Arc Length

1) $L=\int_a^b \sqrt{1+[f'(x)]^2}\,dx$

$f(x)=x^3 \quad f'(x)=3x^2$

$L=\int_0^2 \sqrt{1+9x^4}\,dx$ E

2) $L=\int_1^4 \sqrt{1+9x^4}\,dx$

$[f'(x)]^2 = 9x^4$

$f'(x) = 3x^2$

$f(x) = x^3+C$

$f(1) = 1+C = 6$

$C=5$

$f(x) = x^3+5$ B

3) $y=\cos(2x)$

$y' = -2\sin(2x)$

$L=\int_0^{\pi/4} \sqrt{1+(-2\sin(2x))^2}\,dx$

$= 1.318$ D

4) $x=y^3 \quad \frac{dx}{dy} = 3y^2$

$L=\int_{-2}^2 \sqrt{1+9y^4}\,dy$ C

5) $y=\frac{2}{3}x^{3/2}$

$\frac{dy}{dx} = x^{1/2}$

$L=\int_0^8 \sqrt{1+x}\,dx$

$= \frac{2}{3}(1+x)^{3/2}\Big|_0^8$

$= \frac{2}{3}(9)^{3/2} - \frac{2}{3}$

$= \frac{2}{3}(27) - \frac{2}{3}$

$= \boxed{\frac{52}{3}}$ B

6) $y=x^{2/3}$

$\frac{dy}{dx} = \frac{2}{3}x^{-1/3}$

$L=\int_{-1}^1 \sqrt{1+\left(\frac{2}{3}x^{-1/3}\right)^2}\,dx$

$= \int_{-1}^1 \sqrt{1+\frac{4}{9}x^{-2/3}}\,dx$

OR

$x=y^{3/2} \quad \frac{dx}{dy} = \frac{3}{2}y^{1/2}$

$L = 2\int_0^1 \sqrt{1+\frac{9}{4}y}\,dy$ A

7) $Y_1 = 4x - x^3 + 1 \qquad Y_2 = \frac{3}{4}x$

$Y_1 = Y_2$ @ $x = 1.9404 \to B$

a) $\text{Area} = \int_0^B [Y_1 - Y_2]\,dx$

$= 4.514$

b) WASHER

$V = \pi \int_0^B \left[(Y_1)^2 - (Y_2)^2\right] dx$

$\doteq 57.463$

c) $Y_1(0) = 1$

$L = 1 + \int_0^A \sqrt{1 + (4 - 3x^2)^2}\,dx + \int_0^A \sqrt{1 + \left(\frac{3}{4}\right)^2}\,dx$

8) a) $f_{avg} = \frac{1}{5}\int_0^5 f(x)\,dx$

$= \frac{1}{5}[-10] = \boxed{-2}$

b) $\int_0^{10} (3f(x) + 2)\,dx$

$3\int_0^{10} f(x)dx + \int_0^{10} 2\,dx$

$3[-10 + 27] + 20 = \boxed{71}$

c) $g(x) = \int_5^x f(t)\,dt$

$g'(x) = f(x) = 0$

$x = 5$

− +

0 5 10

g is dec on $(0,5)$

$g''(x) = f'(x) = 0$

$x = 3$ & $x = 8$

− + −

0 3 8 10

g is concave up on $(3, 8)$

g is decreasing & concave up on $(3,5)$ b/c $g'' > 0$ & $g' < 0$.

d) $h = 2f\left(\frac{x}{2}\right) \qquad h'(x) = f'\left(\frac{x}{2}\right)$

$L = \int_0^{20} \sqrt{1 + \left[f'\left(\frac{x}{2}\right)\right]^2}\,dx$

$u = \frac{x}{2} \qquad u(0) = 0$

$du = \frac{1}{2}dx \qquad u(20) = 10$

$2\,du = dx$

$L = 2\int_0^{10} \sqrt{1 + [f'(u)]^2}\,dx$

$= 2(11 + 18) = \boxed{58}$